Title: Maximize Your Serving Size

Brief Overview:

This activity will guide students to discover differences between the volume of cylinders and cones. Students will design and create an ice cream holder model with given specifications.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Geometry
Students will demonstrate their ability to apply geometric relationships to three dimensional objects. They will demonstrate congruency, similarity, symmetry, and reflection and apply these concepts to the solution of geometric problems.

Measurement
Students will demonstrate and apply concepts of measurement using non-standard and standard units and metric and customary units. They will estimate and verify measurements. They also will apply measurement to interdisciplinary and real-world problems solving situations.

Data Analysis and Probability
Students will demonstrate their ability to collect, organize, and display data and will interpret information obtained from displays. They will demonstrate the basic concepts of probability, such as predicting and finding probabilities.

- **Process Standards**

  Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

Links to National Science Education Standards:

- **Unifying Concepts and Processes**
  Students will demonstrate their ability to measure, create, and explain models.

- **Science as Inquiry**
  Students will demonstrate their understanding about scientific inquiry.

- **Physical Science**
  Students will demonstrate properties of objects and materials.

- **Science and Technology**
  Students will demonstrate their abilities and understanding of technological design, science, and technology.

Grade/Level:

Grades 7-8, General Mathematics and Geometry

Duration/Length:

This unit will take approximately four 55-minute math periods.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Basic TI-73 or other graphing calculator pie graph skills
- Basic computer skills for making a spreadsheet
- Measurement
- Geometry terminology
- Construction of 3D shapes (cones, cylinders)

Student Outcomes:

Students will:

- predict and measure the volumes of cylinders and cones.
- apply the formula for the volume of cylinders and cones by using a spreadsheet.
- construct a circle graph using a TI-73 or other graphing calculator.
- design and create a container that holds a specified volume.
- work with other students in a group setting.
- share findings by means of discussions, activities, and final assessment.

Materials/Resources/Printed Materials:

- Teacher information notes
- Cylinder and cone visual examples
- TI-73 or other graphing calculators
- Computer lab equipped with spreadsheet program for volume spreadsheet
- Activity Worksheets 1, 2, 3, and 4
- Final assessment instructions and worksheets for container creations
- Rice
- Metric measuring tapes
- Compass
- Ruler (metric units)
- Ice cream cones (plain cup and sugar types), paper or plastic type cup container that is less than 5 cm x 5 cm size

Development/Procedures:

Pre-Activity Lesson:

- Using the teacher information notes as well as cylinder and cone visuals, the teacher should have had class sessions about volume comparisons of cylinders and cones including use of a calculator for various operations. The students should know the two different formulas to calculate volume for cylinders and cones.

Day 1: "What's the Scoop? -- Scream for Ice Cream!"

- Pass out Activity Sheet 1 and have the students work in pairs to answer the geometry term sheet; go through the class answers together.
- Ask students what flavors of ice cream they like best (an icebreaker activity).
- Ask students what type of container they prefer to have their ice cream served in giving the choices of a cone (plain or sugar) or a container (paper or plastic, whichever the teacher intends to use in the activity). Survey the class and display the results on the chalkboard or overhead.
- Teacher should pass out the sugar cone, plain cone, ice cream container, ruler, measuring tape, rice, and TI-73 or other graphing calculators and Activity Sheet 2. Teacher should go through the directions on Activity Sheet 2 for making a circle graph with the students before students are broken up into groups to make their circle graphs on the TI-73 or other graphing calculators.
- Class discussion on survey results.

Day 2: "Will a Cone or Cylinder Maximize Your Serving Size?"

- Make connections from yesterday's activity to today's activity through discussion.
- Teacher will go over the directions for using the computers and working through the volume of cylinders and cones spreadsheet.
- Students need to have time on computers for working through the Activity 3 Computer Spreadsheet.
- Activity 3 Computer Spreadsheet will be completed during this class period and turned in for evaluation.
- This activity is an individual activity.

Day 3: "Maximize Your Serving Size"

- Teacher will distribute the following materials to the student groups: metric measuring tape, TI-73 or other graphing calculator, compass, ruler, Activity 4 "Maximize Data Sheet, ice cream cones (1 plain and 1 sugar), and ice cream container (plastic or paper).
- Teacher will review the instructions for the Activity 4 Data Sheet for measuring and finding the volume of the different ice cream containers.
- Student groups will have the class period to complete the Activity 4 Data Sheet.
- Activity 4 data sheet will be collected for evaluation.

Day 4: Final Assessment for "Maximize Your Serving Size"

- Teacher will distribute Final Assessment instructions and worksheet and will review with the students.
- In this assessment the students will design an ice cream holder that meets certain specifications.
- Students will sketch and design an ice cream container model and will make calculations for the volume of the model.
- The model and all calculations will be turned in for evaluation.

Performance Assessment:

Students will turn in Activity Sheets 3 and 4 which will be evaluated according to the rubric below. The assessment given on Day 4 will have a separate rubric which is included with the assessment sheet.

Rubric:

3: Any/all calculations are correct. Students can communicate math ideas, processes, and concepts. Student's written responses show clear and logical thinking.

2: Some calculations are correct. Students show some ability to communicate math ideas, processes, and concepts. Student's written responses show some level of logical thinking.

1: No calculations are correct, but effort was demonstrated. Students show little to no ability to communicate math ideas, processes, and concepts. Student's written responses show little to no level of logical thinking.

Extension/Follow Up:

- After the Activity 2 prediction activity, the teacher can use rice to fill the cones or container to compare volume of the cones and cylinder.
- Teacher can use other types of containers or cones to calculate volume, make predictions, and find results.
- Additional ideas can be found at the Internet site, http://www.makeicecream.com.
- Students can change radius and height of containers to investigate effects on volume and then graph results.

Authors:

Linda A. Johnson	Deidra D. Murray	Renee J. Rozman
Crestwood Middle School	Coronado School	St. John's School
Chesapeake, VA	Norfolk, VA	St. Mary's County, MD

Teacher Notes for the Volume Activities and Lesson Introduction

Maximize Your Size:

1) Volume **Formula** for Cylinder

$$V = Bh$$
$$= \pi r^2 h$$
$$\pi = 3.14$$
$B = \pi r^2$ = Area of the circular base
h = Height of the cylinder

2) Volume **Formula** for Cone

$$V = \frac{1}{3} Bh$$

$$= \frac{1}{3} \pi r^2 h$$

- View the above volume formulas.
- The volume of a cone is 1/3 the volume of a cylinder with the same base area (B) and height (h).
- Demonstrate the formula by filling a cone with clear liquid and pouring it into a clear cylinder. Use a cone and a cylinder that are very close in size.

Other Introductory Ideas

- Display the visuals of the cones and cylinders; have a discussion with the students on similarities and differences of the two solids.
- Use the visuals when explaining the formulas.
- Students need to have prerequisite knowledge of geometry terms: height, cone, cylinder, area, volume, diameter, radius, and cubic units.

Reference

Kaplan, Andrew et. al., Math on Call, Great Source Information Group Inc., Wilmington, MA, 1998.

What's the Scoop?

Activity I

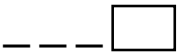
Name: _____ **Score:** _____

Date: _____

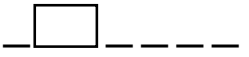
Directions: Using the given clues, print out the word you think matches the clue. Use the letters in the boxes to find the mystery word given at the bottom of the page.

1. The perpendicular distance from the vertex to the base of a cone or between the bases of a cylinder

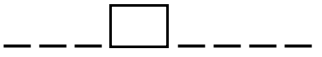
2. A 3-dimensional figure with one curved surface, one flat, usually a circular surface, one curved edge and one vertex



3. The number of cubic units needed to fill a solid figure

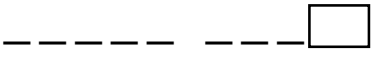


4. A line segment (chord) passing through the center of a circle

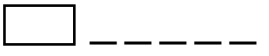


5. The number of square units needed to cover a region

6. The measurement for volume



7. The length of segment from the center of a circle to a point on the circle



8. A 3-dimensional figure with two parallel and congruent circular bases one curved surface, two curved edges and no vertices

Mystery Word Clue: The branch of math that explains properties and relationships of points, angles, surfaces, and solids

Answers

1. height
2. cone
3. volume
4. diameter
5. area
6. cubic unit
7. radius
8. cylinder

Mystery Word - GEOMETRY

SCREAM FOR ICE CREAM!

Activity 2

Name: _____ **Score:** _____
Date: _____

Activity: Your task is to identify shapes used to hold ice cream. Survey the class to determine their favorite ice cream holder. Use a table to collect and record the class data. Use the data to graph the results using a graphing calculator with pie chart/circle graph utilities.

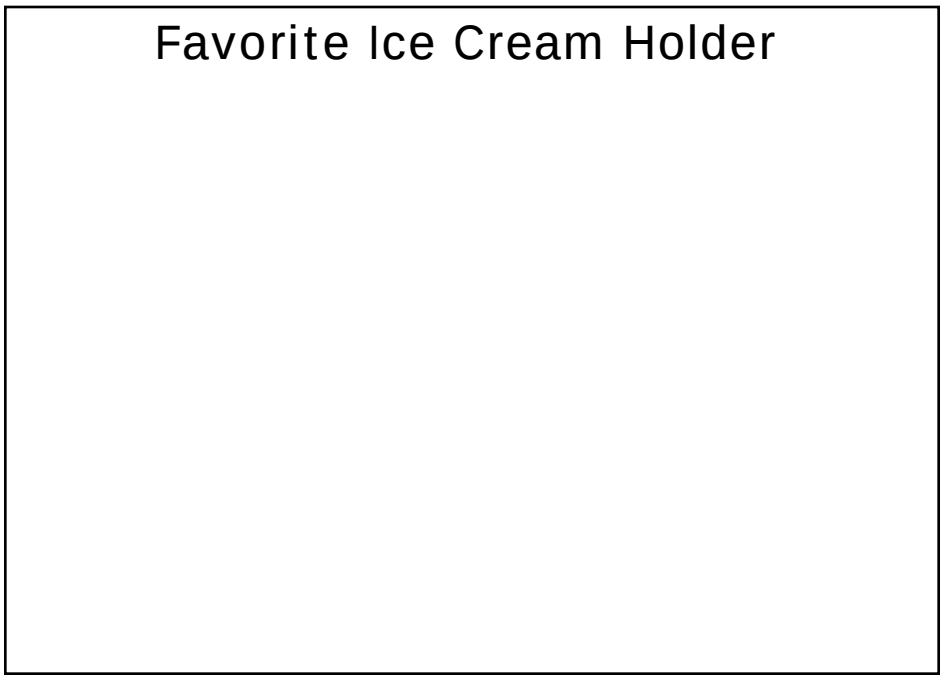
Record the following data in the table:

- the type of holders
- choice of holder with the ice cream

Scream Cream Selections

HOLDERS (L1)	Tally	Frequency (L2)
Type Holder (plain cone)		

Use data from the table to make a circle graph on the TI-73. Make a sketch of your circle graph below.



1. Why do you prefer one holder over another? List reasons for preferences.

_____

_____

2. Predict which shape from your circle graph will hold the most soft serve ice cream.

_____

3. Place the holders in order for the amount of ice cream you predict they will hold. (least to greatest) _____

_____

4. Why do you think one container holds more ice cream than the other? _____

_____

5. Does the shape of the container affect the amount of ice cream it will hold? _____

_____

6. Which shape holds the most soft serve ice cream? Explain the prediction?

_____

_____

_____

Directions for Making a Circle Graph on the TI-73 Calculator

Note:
(Remember different brands of calculators with circle graph or pie chart utilities can be used for this assignment)

1. Display the **LIST** Editor by selecting **LIST** on the calculator.

2. Clear **L1** by selecting the up arrow key, highlighting **L1** and pressing **ENTER.**

3. Enter each type of ice cream holder listed under **L1.**

4. Enter the holder's name by:

   - Selecting the **(yellow) 2nd** key.
   - Selecting the Math button directly underneath the **2nd** key (the text display menu appears).
   - Place each word between the quotation marks, after each letter press enter but use the cursor key to select the needed letters (example - "sugar").
   - When the word is entered, check for quotation marks, select **DONE** using the up arrow key and press **ENTER** twice.
   - The word should appear under **L1** in the **LIST** column.
   - Repeat step 4 to complete each **LIST.**

5. Enter the data by:

   - Using the arrow keys to move to **L2.**
   - Enter the number of students that selected each type of holder as their favorite in **L2 (List 2).**

6. Make the circle graph by:

   - Selecting the **(yellow) 2nd** key and the **PLOT/(Y=)** key.
   - **STAT PLOTS** appear on the screen.
   - Select **Plot1**, press the **ENTER** key, and with the cursor blinking on the word **ON**, select by pressing **ENTER.**
   - Arrow down and over using the down and right arrow keys, selecting the pie chart/circle graph illustration and press **ENTER.**
   - Use the arrow key to the **CategList**, select **(yellow key) 2nd key** and the **STAT** key, select **L1** and press **ENTER.**
   - Use the arrow key, select **Data List**, press the **(yellow) 2nd** key and the **STAT** key, select **L2** by arrowing down and press the **ENTER** key.
   - Use the arrow key, select **Percent** and press the **ENTER** key.

7. Display the circle graph by:

   - Make sure all of the other plots are **OFF.**
   - Press the **GRAPH** key in the top row.
   - Circle graph will appear on the screen.
   - Press the **TRACE** key and use the arrow keys to identify each section of the graph.

Name: _____ **Score:** _____
Date: _____

Will a Cone or Cylinder Maximize Your Serving Size?

Activity 3

Use a spreadsheet to discover the ratio between the volume of a cone and a cylinder. A spreadsheet is useful to perform calculations quickly and enables the student to look for patterns and relationships in the numbers. The student will use the spreadsheet to compute the volume of cones and cylinders. The last column of your spreadsheet will compare these volumes.

1.) Enter the headings and number data given below in your spreadsheet.

	Radius	Height	Volume of Cone	Volume of Cylinder	Ratio of Cone and Cylinder
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

2.) Find the volume of the cone. Enter the formula in cell **D2** =1/3*PI( )*B2^2*C2 .

3.) Now, use the spreadsheet **Fill Down** command in the **Calculate** menu to fill in the formula for the remaining cell in column **D**. Highlight cells **D2-D11**. Click on the **Calculate** menu. **Click on Fill Down.**

4.) Find the volume of a cylinder. Enter the formula in cell **E2** =PI( )*B2^2*C2 .

5.) **Fill Down** the formula for the volume of a cylinder as in step 3.

6.) Compute the ratio of the volume of a cone to the volume of a cylinder given the same radius and height in cell **F**.

7.) Enter the formula for the ratio in cell **F2** =D2/E2.

8.) **Fill Down** the formula for the ratio of the volumes as in step 3.

9.) Insert 10 values for radius and the height. Observe the calculations for the volume of each shape.

10.) Continue entering values until you can answer question #11.

11.) Explain the connection between the 1/3 in the formula for the volume of a cone, the ratio of the volume of the cone, and the volume of the cylinder.

_____

_____

_____

_____

_____

_____

_____

_____

12.) Insert a footer and type your name.

Teacher Sample Spreadsheet for Activity 3

	Radius	Height	Volume of Cone	Volume of Cylinder	Ratio of Cone and Cylinder
1	10	20	2094.40	6283.19	0.33333333333333
2	20	30	12566.37	37699.11	0.33333333333333
3	30	40	37699.11	113097.34	0.33333333333333
4	40	50	83775.80	251327.41	0.33333333333333
5	50	60	157079.63	471238.90	0.33333333333333
6	60	70	263893.78	791681.35	0.33333333333333
7	1	2	2.09	6.28	0.33333333333333
8	2	3	12.57	37.70	0.33333333333333
9	3	4	37.70	113.10	0.33333333333333
10	4	5	83.78	251.33	0.33333333333333

Name: _____ **Score:** _____
Date: _____

Maximize Your Serving Size Data Sheet

Activity 4

Use this data sheet as a guide to find the ice cream holder that gives you the greatest serving size.

Materials needed:

- Sugar Cone
- Plain Cone
- Cup
- Measuring Tape
- Ruler
- Calculator

1.) Measure the diameter and height of the three shapes. Use centimeters for your measurements

2.) Sketch the three shapes below and include the dimensions.

Sugar Cone	Plain Cone	Cup

3.) Using the formula for volume, calculate the volume of each shape. Record the calculations below. Include the correct units.

Sugar Cone	Plain Cone	Cup

4.) Which shape has the greatest volume? Why? _____

_____

_____

_____

_____

5.) Compare your group calculations with other groups.

Calculate the mean for all volumes

Volume of Sugar	Volume of Plain Cone	Volume of Cup
Mean Volume	Mean Volume	Mean Volume

6.) How does the volume calculations compare to the estimation at the beginning of the unit?

_____

_____

Teacher Notes

(for Activity 4, "Maximize Your Serving Size Data Sheet)

Calculations for the measurement of sample plain cone, sugar cone, and cup holders. (cone sizes may vary)

Sugar Cone Sample:
Radius: 2 cm
Height: 11 1/2 cm
Volume: 48.17 cubic centimeters

Plain Cone Sample (2 cylinders were used to estimate the volume):
Radius 1: 2.25 cm
Height 1: 2.3 cm
Volume 1: 36.58 cubic centimeters
Radius 2: 1.5 cm
Height 2: 5.6 cm
Volume 2: 39.58 cubic centimeters
Total Volume= Volume 1 + Volume 2 = 36.58 + 39.58 = 76.16 cubic centimeters

Cup Sample
Radius: 5 cm
Height: 5 1/2 cm
Volume: 431.97 cubic centimeters

******A smaller cup can be used to make the volumes more compatible and more challenging to compare.

Assessment for Maximize Your Serving Size

Name: _____ **Score:** _____

Date: _____

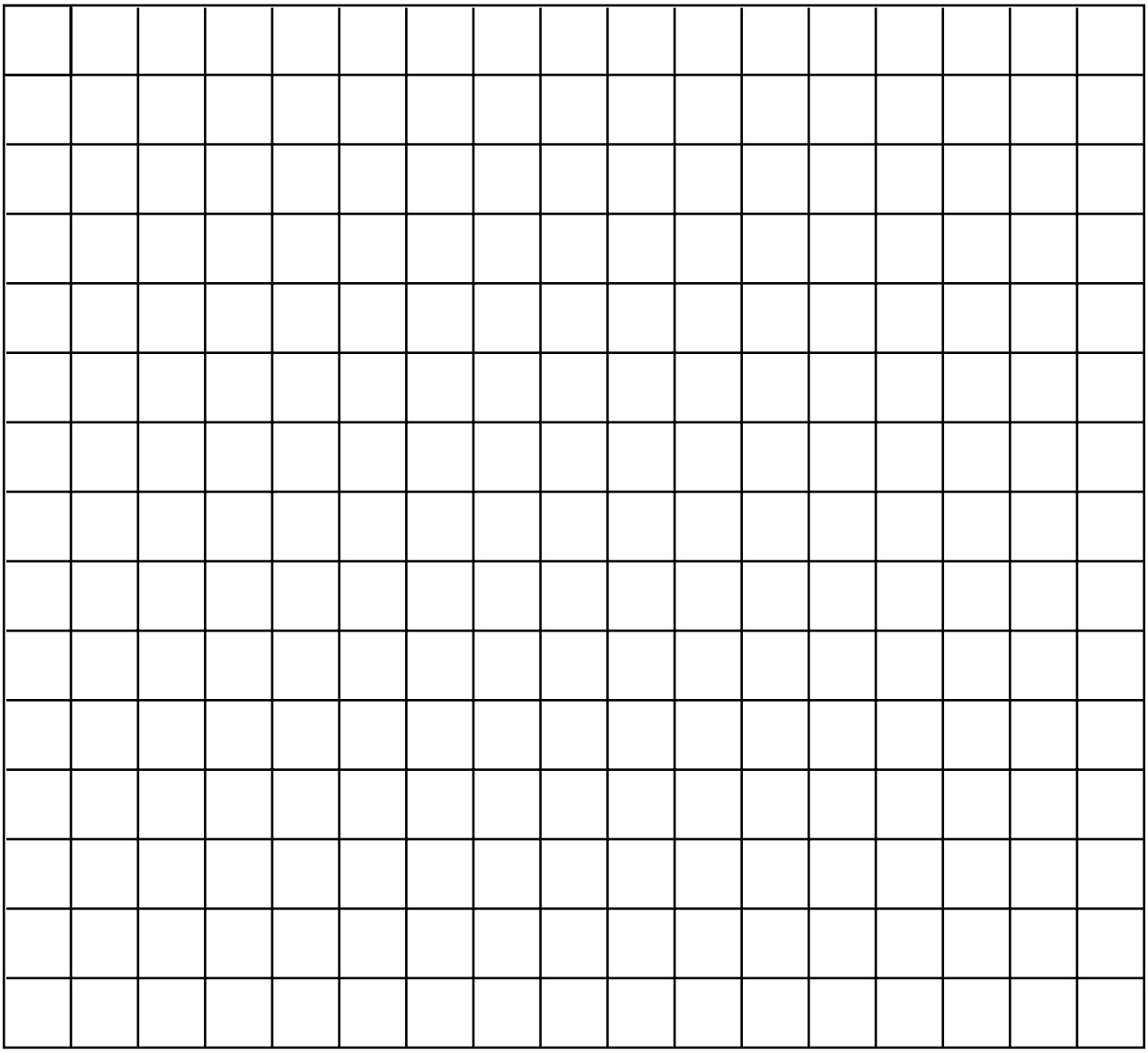
Design an ice cream holder using cones and cylinders as your guide. This holder **must** have a volume greater than **200** cubic centimeters and less than **250** cubic centimeters. Sketch your design and use the needed formulas to show your calculations. Use your sketch and the additional graph paper to make a model out of your design.

Sketch:

Calculations (Show All Work):

Rubric For Maximize Your Serving Size

4	Volume is within specifications of >200 and <250. All calculations are correct. Correct formulas are used for shape design. Sketch of holder is neat and dimensions are labeled. Correct dimensions are used for paper model. All work is shown.
3	Volume is close to specifications. All calculations are correct. Correct formulas are used for shape design. Sketch of holder is neat and dimensions are labeled. Minor errors in dimensions for paper model. Most of work shown.
2	Volume is close to specifications. Most calculations are correct. Correct formulas are used for shape design. Sketch of holder is neat but not labeled. Errors in dimensions for paper model. Some work shown.
1	Volume is not close to specifications. Few calculations are correct. Some of the correct formulas are used for shape design. Sketch of holder is not clear. Errors in dimensions for paper model. Little work shown.
0	Volume is not close to specifications. No calculations are correct. No formulas are correct. Sketch of holder is not clear. No paper model. No work shown.



Teacher Sample Assessment Calculations:

Cup or Cylinder

Dimensions of 2.5 cm for the radius and 11 cm for the height will give a volume of about 216 cubic centimeters.

Cone

Dimensions of 4.5 cm for the radius and 10 cm for the height will give a volume of about 212 cubic centimeters.